Simplified model for heat transfer and solidification in continuous casting.*

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A simplified model is presented for predicting the solid fraction at the mold exit and the liquid pool depth from readily measurable parameters in continuous casting operations. The results obtained by this model are satisfactory when compared to experimental values and results obtained with more complex model representations. The relative simplicity of the mathematical calculations required for the model suggest its utility as the basis for control functions in plant operation.

INTRODUCTION

Cooling and solidification in continuous casting occur principally in the mold section and in the secondary cooling section. The cooling that takes place in the mold must sufficient to cool the liquid metal to the point where a solid layer is formed on the surface of sufficient thickness to mechanically support the pool of liquid metal inside the casting as it leaves the mold. The secondary cooling process, on the other hand, must be sufficient to complete the solidification process initiated in the mold. The cooling velocity is a critical parameter for other reasons as well: an excessively large cooling rate tends to produce cracks in the bending zone of the casting but an excessively slow cooling rate results in an exceedingly long liquid pool. The continuous casting process is illustrated schematically in Fig. 1.

The process of heat transfer in the mold from the liquid metal to the cooling water circulating through the mold takes place by a sequence of steps of different mechanisms: convection and conduction inside the liquid metal, conduction through the solid metal layer, convection, conduction and radiative heat transfer across the gap between the solid metal layer and the mold wall, conduction through the mold wall, and finally, convective heat transfer from the mold to the cooling water.

Several mathematical models have been proposed for calculating the thickness of the solid metal layer formed in the mold [1, 2 and 3]. These models, using various degrees of mathematical sophistication for dealing with the heat transfer and heat balance inside the mold depend, nevertheless, for the final results on the estimation of a heat transfer coefficient between the surface of the casting and the mold wall. The estimation of this transfer coefficient, with any degree of accuracy, is a practically impossible task since many variables come into play in determining its value: the formation of an air gap of variable width between the metal and the mold [4] type and thickness of lubricant, radiative heat transfer across the gap, the effect of casting velocity on the aforementioned parameters, etc. In any case, an average heat transfer coefficient is generally used in the mathematical models, although experimental data and theoretical considerations would indicate that the heat transfer coefficient changes along the length of the mold section.

The heat transfer in the secondary cooling section is even more complex due to the irregular geometry and various types of transfer processes taking place at different points of the section. As is illustrated in figure 2, the secondary cooling section consists of different areas where heat transfer takes place by contact with the pinch rolls, radiation, and convective heat transfer to impinging jets or water pools accumulated in the proximity of the rolls. The heat transfer coefficient to impinging water jets alone is a complex function of jet velocity, flow rate, surface temperature and geometry of the system (as is illustrated in figure 3). In view of the other concurrent heat

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Fig. 2. The various mechanisms of heat transfer in the secondary cooling zone [7]. (Percentages shown refer to estimated contribution of each mechanism to the overall heat transfer).

transfer mechanisms the resulting overall heat transfer coefficient is practically impossible to describe analytically in terms of process variables.

From the purely operative point of view there are two process variables that can be monitored with relative ease and are related, by means of some simplifying assumptions, to the heat transfer coefficients inside the mold and in the secondary cooling section: in the mold, the cooling water flow rate and its initial and final temperatures (i.e. the enthalpy increment of the cooling water); in the secondary cooling section the average temperature of the outside surface of the casting (“wall temperature”) which, as experimental results indicate, is a non-monotonic variable along the secondary cooling section with values that fall within ±50°C along the section (e.g. as illustrate in figure 4).

In the present paper a simplified model has been developed which permits the calculation of the fraction of metal solidified at the exit from the mold and the liquid pool depth in terms of the aforementioned parameters and other operational variables and material properties.

The main assumptions and approximations involved in this formulation are the following:

i. The liquid metal is at its melting point at the entrance to the mold and throughout the length of the fractionally solidified ingot.

ii. The heat transfer in the mold section may be represented by a single global heat transfer coefficient derived from the temperature change of the cooling water and its flow rate.

iii. The solidified fraction of metal is represented as a linear function of distance both inside and in the secondary cooling section. Similarly the average temperature of the solid metal in the fractionally solidified ingot is assumed to be the mean between the liquid temperature and the wall temperature.

iv. The thickness of the solidified layer is assumed to be uniform around the perimeter of the casting.

v. The heat transfer in the secondary cooling section is also represented by an effective global

![Graph of heat transfer coefficient h as a function of water flow rate per unit area V/A. Each curve represents a different arrangement.](image)
heat transfer coefficient estimated from a wall temperature of the metal averaged over the length of the secondary cooling section.

The results obtained compared favorably with those of other mathematical models [2, 3] and agree well with experimental results obtained by Morton and Weinberg [5] and continuous casting plant data. The simplicity of the mathematical relationships between the process variables resulting from the model and its direct relationship to readily measurable and controllable process variables would make it a useful tool for control of continuous casting processes.

2. Thermal balance in the mold

The thermal energy contained in the liquid metal entering a continuous casting mold consists of sensible heat (relative to some reference temperature) and the heat of melting. Thus the enthalpy per unit volume of liquid metal at the melting point may be represented as:

\[ H'_V = \frac{H_1(T_m) - H_o}{V} = \rho_s C_p (T_f - T_o) + H_f \]  

where

- \( \rho_s \) = density of the solid metal (Kg · m\(^{-3}\))
- \( C_p \) = average specific heat of the solid (JKg\(^{-1}\)K\(^{-1}\))
- \( T_f \) = normal melting temperature (K)
- \( H_f \) = specific heat of melting (J · Kg\(^{-1}\))
- \( T_o \) = reference temperature (e.g., that of the cooling medium) (K)

In an analogous manner the enthalpy content per unit volume of solid metal at the point where the last liquid solidifies (see figure 4) may be written as

\[ H'_V = \frac{H_2(T_m) - H_o}{V} = \rho_s C_p (T_2 - T_o) \]  

where \( T_2 \) is the average temperature of the solid at the point of complete solidification \( (T_o < T_2 < T_f) \). The difference between \( H_1(T_m) \) and \( H_2(T_m) \) is the amount of heat removed per unit volume between the entrance to the mold and the point of complete solidification. Denoting this quantity by \( H'_V \):

\[ H'_V = H_1(T_m) - H_2(T_m) = \rho_s C_p (T_f - T_2) + H_f \]  

The heat removal in the continuous casting process takes place in two sequential steps (see figure 1): first heat is removed as the metal passes through the mold cooled by circulating water; subsequently cooling is carried out in the so-called “secondary cooling section” where the metal is exposed to direct contact with impinging water jets.

The mold section is relatively short and typically only about 15% of the total heat is removed in this section. Since the heat transfer process is different in each of the sections it is found convenient to handle their analytical representation separately.

Although only a small fraction of the total heat is removed in the mold section, the control of this heat transfer is highly important since it determines the thickness of the solid layer formed and hence the mechanical resistance of the slab or billet as it leaves the mold. The definition of a heat transfer coefficient between the casting and the mold wall is complicated by the fact that, as the metal solidifies and shrinks, an air gap of variable width is formed between the casting and the mold [4 and 6]. Also, the simultaneous heat transfer by conduction through metal - to - metal contact points and by radiation further complicate the estimation of an effective heat transfer coefficient. The experimental data and estimates by various investigators show that this heat transfer coefficient decreases by 30 to 50% between the ent-
rance to and exit from the mold [7]. However, as is shown below, the overall or global heat transfer coefficient can in practice be replaced by the temperature difference between the water entering and leaving the mold and its flow rate. Thus relations between solid shell thickness and readily measurable process parameters can be established.

For metal which has travelled for time \( t \) through the mold the heat removed, per unit volume of metal, may be expressed in terms of a global heat transfer coefficient \( h_1 \) as,

\[
H_i(t) = \int_0^t A_v h_1(T_f - T_{ave}) dt \tag{4}
\]

where

\[
A_v = \text{heat transfer area per unit volume of metal (m}^2\text{)}
\]

\[
T_{ave} = \text{average temperature of the cooling water circulating through the mold (K)}
\]

and it is assumed that for the time considered liquid metal exists at the center of the mold, i.e. the solidification is only fractional. For such partially solidified metal the heat content per unit volume may be written in terms of solidified fraction \( f_s(t) \) as:

\[
H^{(1)}(t) = \rho_s C_p \left[ T(t) - T_i \right] f_s(t) + \rho_s C_p \left[ T_f - T_i \right] f_s(t) + H_i \left[ 1 - f_s(t) \right] \tag{5}
\]

where \( T(t) \) represents the average temperature of the solid formed after time ‘t’ in the mold.

The difference between the heat contents given by equations (1) and (5) may be equated to the heat transferred according to equation (4):

\[
H^{(1)}(t) - H_i(t) = f_s(t) \rho_s \left[ C_p \left( T_f - T_i \right) + H_i \right]
\]

Thus, solving for the solidified fraction \( f_s(t) \):

\[
f_s(t) = \frac{H_i(t)}{\rho_s \left[ C_p \left( T_f - T_i \right) + H_i \right]} \tag{7}
\]

The average temperature, \( T(t) \), of the solid may be expected to be a complex function of the geometry of the system and various process parameters [1]. In order to obtain a simple, functional relationship between the residence time (or position) if the casting velocity is constant) inside the mold. The global heat transfer coefficient inside the mold may be related, on the other hand, to the flow rate of the cooling water and its inlet and outlet temperatures. The rate of heat removal, \( \dot{Q} \), through the mold is:

\[
\dot{Q} = \dot{m} C_w \left( T_{out} - T_{in} \right) \tag{8}
\]

where

\[
\dot{m} = \text{mass flow rate of water (Kg s}^{-1}\text{)}
\]

\[
C_w = \text{specific heat of water (J Kg}^{-1}\text{K}^{-1})
\]

\[
T_{out}, T_{in} = \text{outlet and inlet temperatures, respectively of the cooling water (K)}
\]

The average global heat transfer coefficient inside the mold may be evaluated as:

\[
h_1 = \frac{\dot{Q}}{A_t \left( T_f - T_{ave} \right)} \tag{9}
\]

where \( A_t \) is the total heat transfer area between the casting and the mold wall and \( T_{ave} = 0.5 \left( T_{out} + T_{in} \right) \). Substituting this average heat transfer coefficient into equation (4) and combining equation (7).

\[
f_s(t) = \frac{\dot{Q} t}{V \rho_s \left[ C_p \left( T_f - T(t) \right) + H_i \right]} \tag{10a}
\]

where \( V = A_t / A_v \) is the total volume of metal inside the mold. Introducing the casting velocity ‘u’ (\( u = dz/dt \)) the solidified fraction may be expressed in terms of position ‘z’ within the mold:

\[
f_s(z) = \frac{\dot{Q} z}{V u \rho_s \left[ C_p \left( T_f - T(z) \right) + H_i \right]} = \frac{\dot{Q} z}{S u \rho_s \left[ C_p \left( T_f - T(z) \right) + H_i \right]} \tag{10b}
\]

where \( S \) and \( l \) are the cross sectional area and length, respectively, of the mold.

Some further simplification is needed in order to estimate the average temperature. Assuming that the average temperature \( T(z) \) is a mean between the melting temperature and the wall temperature of the cas-
ting $T_w$, the heat transfer rate across the solid shell may be related to the thermal conductivity of the solid, i.e.

$$\dot{Q} \approx K_s A_s \frac{T_f - T_w}{\Delta X_s}$$  \hspace{1cm} (11)$$

where

- $K_s = \text{thermal conductivity of the solid (Jm}^{-1}\text{s}^{-1}K^{-1})$
- $\Delta X_s = \text{average thickness of solid layer (m)}$
- $\bar{T_w} = \text{average skin temperature of the casting inside the mold (K)}$

Under the assumption of a linear variation of solid temperature with position inside the solid shell of the casting, with $T_w(l)$ representing the skin temperature at the exit from the mold ($z = l$):

$$\bar{T_w} \approx 0.5[T_f + T_w(l)] \hspace{1cm} (12a)$$

and

$$\bar{T}(l) \approx 0.5[T_f + T_w(l)] \hspace{1cm} (12b)$$

thus

$$T_f - \bar{T}_w \approx T_f - \bar{T}(l) \hspace{1cm} (12c)$$

Similarly, under the assumption of a linear increase with distance 'z' of the thickness of the solidified layer:

$$\Delta X_s \approx 0.5 \Delta X_s(z = l) \hspace{1cm} (13)$$

Substituting equations (12c) and (13) into equation (11) and solving for $T_f - \bar{T}(l)$:

$$T_f - \bar{T}(l) \approx T_f - \bar{T}_w = \frac{\dot{Q} \Delta X_s(z = l)}{2K_s A_s} \hspace{1cm} (14)$$

The thickness of the solidified layer may, in turn, be expressed in terms of the solidified fraction, $f_s$. Assuming a uniform thickness of the layer and a regular shape casting (square or cylindrical section):

$$\Delta X_s(z) = 1 - \left[1 - f_s(z)\right]^{1/2} \hspace{1cm} (15)$$

where $L$ is the characteristic dimension of the casting (radius in the case of a cylinder, or one half of the edge for a slab).

Combining equations (14) and (15) with equation (10b), evaluated for $z = l$ to eliminate $\Delta X_s(l)$ and $[T_f - \bar{T}(l)]$:

$$[1 - f_s(l)]^{1/2} = \left[1 - \frac{2K_s A_s}{(S\rho_s C_p S) f_s(l) + \frac{H_s K_s A_s}{C_p Q L}} \right] \hspace{1cm} (16)$$

This equation may be solved for $f_s(l)$ by trial and error or as a cubic equation in $f_s(l)$, since all other parameters appearing in the equation are material properties ($K_s, \rho_s, C_p, H_s)$ or operational variables ($A_t, S, u, L, Q$).

The results of such calculations are compared to the experimentally measured [12] solid layer thickness in figure 5. This same figure also shows the calculated results based on the more detailed and sophisticated models of Hills [2] (integral profile method) and Mizikar [3] (finite difference model). In view of the satisfactory results obtained by equation (16) it could be used as a relation for control of continuous casting processes.

3. Heat balance for secondary cooling

The heat extraction in the secondary cooling process (see figure 2) occurs primarily by direct contact of water (or steam) with the metal surface. Consequently three types of metal-coolant contact may result [9]: wet wall, dry wall and an intermediate case. Of these the wet wall contact is generally the most efficient [9], i.e. exhibits the highest effective heat transfer coefficient. For secondary cooling the temperature of the water after contacting the metal can not be readily measured and hence it becomes necessary to use empirical data to estimate the heat transfer coefficient for any particular operational set-up.

The overall rate of heat transfer from the molten metal inside the casting to the cooling water, per unit volume metal, in the secondary cooling process may be written as:

$$\dot{Q} = h_2 (T_f - T_{in}) \frac{A_{12}}{V} = h_2 (T_f - T_{in}) \rho \Delta z \hspace{1cm} (18)$$

where

$$h_2 = \text{global heat transfer coefficient for the secondary cooling process (J} \text{s}^{-1} \text{m}^{-2} \text{K}^{-1})$$
The heat content, per unit volume of metal, relative to the initial temperature of the cooling water, at the entrance to the secondary cooling section is equal to the heat content of the material leaving the mold:

\[ \text{heat content} = \{1 - f_s(l)\} \rho_s [C_p(T_f - T_{in}) + H_f] + f_s(l) \rho_s C_p(T_f - T_{in}) \]  \hspace{1cm} (19)

where \( f_s(l) \) may be calculated by equation (17a).

Similarly, at a position \( z \) beyond the entrance to the secondary cooling section but before the metal is totally solidified \((f_s(z) < 1)\), this same quantity may be calculated as:

\[ \text{heat content} = \{1 - f_s(z)\} \rho_s [C_p(T_f - T_{in}) + H_f] + f_s(z) \rho_s C_p(T_f - T_{in}) \]  \hspace{1cm} (20)

Balancing the difference in heat contents, as given by equations (20) and (19), against the heat transfer rate given by equation (18):

\[ \frac{Q}{Su} = \frac{h_2(T_f - T_{in})p\Delta z}{Su} = \left[ f_s(z) - f_s(l) \right] \rho_s [C_p(T_f - T_{in})] + H_f [ + f_s(z) \rho_s C_p(T_f - T_{in}) - T_{in}] \]  \hspace{1cm} (21)

solving this equation for \( f_s(z) \)

\[ f_s(z) = f_s(l) \frac{C_p(T_f - T_{in}) + H_f}{C_p(T_f - T_{z}) + H_f} + \frac{h_2(T_f - T_{in})p\Delta z}{\rho_s \left[ C_p(T_f - T_{z}) + H_f \right]} uS \]  \hspace{1cm} (22a)

The factor multiplying \( f_s(l) \) in equation (22) is estimated to have a value of 1.00 ± 0.03 for most operating conditions and hence may be approximated to unity. Thus

\[ f_s \approx f_s(l) + \frac{h_2(T_f - T_{in})}{\rho_s \left[ C_p(T_f - T_{z}) + H_f \right]} \]  \hspace{1cm} (22b)

This equation is applicable to positions \( z \) prior to total solidification, hence the average temperature may again be approximated by the mean of the melting temperature and the wall temperature \([T(z) \approx \frac{1}{2}(T_f + T_w(z))]\), hence:

\[ f_s(z) \approx f_s(l) + \frac{h_2(T_f - T_{in})}{\rho_s \left[ 0.5C_p(T_f - T_w(z)) + H_f \right]} \]  \hspace{1cm} (22c)

In equation (22c) the only unknown parameter is the global heat transfer coefficient \( h_2 \). The wall temperature \( T_w \) could be measured experimentally. In view of the relatively high thermal conductivity of the metal it is possible to estimate the global heat transfer coefficient in terms of the thermal conductivity of the metal assuming a steady state in regard to heat transfer, and an approximately constant transfer area, i.e.:

\[ h_2 \approx \frac{K_p}{\Delta X_s \left( T_f - T_w(z) \right)} \]  \hspace{1cm} (23)

Substituting this relation into equation (22c) and expressing the solid layer thickness in terms of the solidified fraction [according to equation (15)], the following equation is obtained:

\[ [f_s(z) - f_s(l)] \left[ 1 - \frac{1}{2} f_s(z) \right] = F(T_w) \]  \hspace{1cm} (24)

where

\[ F(T_w) = \frac{K_p \Delta z}{L u S \rho_s \left[ T_f - T_{in} \right] \left[ 0.5C_p(T_f - T_w(z)) + H_f \right]} \]  \hspace{1cm} (25)

Equation (24), solved for the condition \( f_s(z) = 1 \), permits the estimation of the liquid pool depth \( (\Delta z) \) inside the casting.

\[ \Delta z \approx \frac{1-f_s(l)}{L u S \rho_s \left[ 0.5C_p(T_f - T_w(z)) + H_f \right]} \left[ T_f - T_{in} \right] K_p \]  \hspace{1cm} (26)

In this equation all parameters on the right hand side are materials properties or operational variables. The wall temperature \( T_w \) becomes the measure of the
effectiveness of the cooling process (as \( Q \) is in the case of cooling in the mold). Experimental evidence [7] indicates that, due to the irregular nature of the secondary cooling arrangement, the wall temperature also varies non-monotonically along this cooling section but its value does not vary by more than ±50°C from an average temperature.

A relationship of the form of equation (26) may thus be used to monitor the structure of the casting (i.e. liquid pool depth) in terms of a measurable parameter (wall temperature), which may also be regarded as an operational variable since it represents the effectiveness of the secondary cooling process.

Equations (16) and (26) may be used to calculate the solid fraction at the exit from the mold \((f_s(l))\) and the depth of the pool of molten metal beyond the mold \((\Delta z)\), respectively. The total pool depth is, of course the sum of the mold length and \(\Delta z\), i.e. \(\Delta z = l + \Delta z\). Figure 5 shows a comparison of the solidified fraction at the mold exit \(f_s(l)\), as calculated by equation (16) and the experimental values of Morton and Weinberg [5]. Included on this same diagram are the values calculated by the "finite difference model" of Mizikar [3] and the "integral profile method" of Hills [2] as reported by Brimacombe and Weinberg [8]. Two results are interesting to note on this diagram. The agreement with experimental data obtained with the simplified model appears to be slightly better than that of other two models. Since the mathematical treatment of the heat balance and conduction process is much more complete and exact in the other model, the results may only be ascribed to the relative effectiveness of representing the global heat transfer process as in the simplified model. Secondly the dispersion of the points in figure 5 is slightly less than in the other models, which, again, can only be attributed to the predominance of the importance of the heat transfer coefficient estimation for the overall representation of heat transfer and solidification processes. This leads to the conclusion that equation (16) is essentially as good, and considerably simpler, than alternate, more complex models, available for representing the same phenomenon.

Figure 6 shows equation (16) plotted for the particular case of a mold 51 cm long, a cross section of 14 cm × 14 cm and a carbon steel of 0.3% C (melting point \(\sim 1448°\) C). This figure shows the relative importance of the cooling rate and casting velocity on the fraction solidified at the exit from the mold. The shaded area in this figure represents the typical range of values of cooling rate and solidified fraction in plant operation [5]. As can be seen, the values predicted by equation (16) coincide satisfactorily with those obtained in practice.

The depth of the liquid pool, as calculated by equation (26), is compared to experimentally measured values [5] in figure 7. In order to apply equation (26) the average wall temperature must be known. This is not the case for the pool depth data used since Morton and Weinberg do not report this parameter for their experiments. An assumed temperature of 1100° C was used which is a typical average value reported in other operations (see figure 3). Again, a reasonable agreement between predicted and expe-
Experimental values is obtained, especially in view of the inaccuracies inherent in the experimental measurements [5] and the necessary simplifications in the model.

Equation (26) is plotted in figure 8 for the case of a square (14 cm x 14 cm) billet of 0.3% C steel and a solidified fraction \((f_s/l)\) at the entrance to the secondary cooling section of 0.26. The shaded section covers the experimental value range reported by Morton and Weinberg [5]. The nearly-linear relationship between average wall temperature and the liquid pool depth is interesting to note.

Estimated solidified fraction and wall thickness at the mold exit and liquid pool depth were also calculated on the basis of operational parameters of the continuous casting plant of SIDOR in Matanzas, Venezuela. Table 1 summarizes a particular set of operational variables and the resulting values of \(f_s/l\), \(\Delta X/l\) and \(l_p\) calculated according to equations (16), (15) and (26), respectively. Although experimental values for these parameters could not be obtained for this particular operation, the calculated values fall within the range of the specifications for the plant [10].

### SUMMARY

In conclusion, it is found that a simplified representation of the heat transfer and solidification process may be used to calculate and/or monitor two important variables in continuous casting: the fraction solidified at the exit from the mold and the depth of the liquid pool. The operational parameters to be controlled or monitored for this purpose are the inlet and outlet temperatures and flow rate of the cooling water in the mold and the average surface temperature of the casting in the secondary cooling stage.

### Table I

**Operating data from SIDOR continuous casting plant steel properties and calculated process parameters.**

**Operating data:**

- Slab section (S) = 0.220 m² (0.175 m x 1.260 m).
- Half thickness (L) = 0.0875 m.
- Heat transfer area in mold (Aₜ) = 1.72 m².
- Temperature change of cooling water in mold: \((T_{out} - T_{in}) = 6.1 \, K\).
- Average inlet temperature for secondary cooling water: \(T_{in} = 303 \, K\).
- Average slab surface temperature in secondary cooling section (est.) \(T_w = 1373 \, K\).

**Steel data:**

- Composition: 0.8% C, 0.10% Si, < 0.020% P, <0.020% S, 0.05% Al.
- Density \((\rho_s)\): 7400 kg · m⁻³.
- Melting temperature, \((T_m) = 1753 \, K\).
- Heat of fusion \((H_f) = 2 \cdot 72 \cdot 10^{-3} \, J \cdot kg^{-1}\).
- Specific heat \((C_p) = 668 \, J \cdot kg^{-1} \cdot K^{-1}\).
- Conductivity \((K) = 29.3 \, W \cdot m^{-1} \cdot K^{-1}\).

**Calculated parameters:**

- Solid fraction at mold exit \(f_s/(l) = 0.16\).
- Solid layer thickness \(\Delta X/(l) = 1 \cdot 88 \cdot 10^{-2} \, m\).
- Liquid pool depth \(l_p = 12.8 \, m\).
### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>$A_t$</td>
<td>total heat transfer area between the casting and the mold wall ($m^2$).</td>
</tr>
<tr>
<td>$A_{t2}$</td>
<td>heat transfer area in the secondary cooling zone ($m^2$).</td>
</tr>
<tr>
<td>$A_v$</td>
<td>heat transfer area per unit volume of metal ($m^2$).</td>
</tr>
<tr>
<td>$C_p$</td>
<td>average specific heat of the solid ($J \cdot kg^{-1} \cdot K^{-1}$).</td>
</tr>
<tr>
<td>$C_w$</td>
<td>specific heat of water ($J \cdot Kg^{-1} \cdot K^{-1}$).</td>
</tr>
<tr>
<td>$f_s(t)$</td>
<td>solid fraction of metal at the mold (dimensionless).</td>
</tr>
<tr>
<td>$H^{(1)}$</td>
<td>enthalpy of liquid metal at the mold entrance ($J$).</td>
</tr>
<tr>
<td>$h_1$</td>
<td>global heat transfer coefficient in the mold ($W \cdot m^{-2} \cdot K^{-1}$).</td>
</tr>
<tr>
<td>$H^{(2)}$</td>
<td>enthalpy of metal at the point where the last liquid solidifies ($J$).</td>
</tr>
<tr>
<td>$h_2$</td>
<td>global heat transfer coefficient for the secondary cooling process ($W \cdot m^{-2} \cdot K^{-1}$).</td>
</tr>
<tr>
<td>$H_f$</td>
<td>heat of melting ($J \cdot Kg^{-1}$).</td>
</tr>
<tr>
<td>$H_0$</td>
<td>enthalpy of metal at reference temperature ($J$).</td>
</tr>
<tr>
<td>$H_{si}$</td>
<td>heat content per unit volume of metal at the entrance of the secondary cooling section ($J \cdot m^{-3}$).</td>
</tr>
<tr>
<td>$H_s$</td>
<td>heat content per unit volume of metal at position $z$ in the secondary cooling section ($J \cdot m^{-3}$).</td>
</tr>
<tr>
<td>$H_v^{(1)}$</td>
<td>enthalpy per unit volume of liquid metal ($J \cdot m^{-3}$).</td>
</tr>
<tr>
<td>$H_v^{(2)}$</td>
<td>enthalpy of solid metal at the point where the last liquid solidifies ($J \cdot m^{-3}$).</td>
</tr>
<tr>
<td>$H_v(t)$</td>
<td>heat removed in a time $t$ in the mold ($J \cdot m^{-3}$).</td>
</tr>
<tr>
<td>$H_v^{(6)}$</td>
<td>heat removed from metal for complete solidification ($J$).</td>
</tr>
<tr>
<td>$H_v^{(9)}$</td>
<td>heat content per unit volume of metal at the mold ($J \cdot m^{-3}$).</td>
</tr>
<tr>
<td>$K_s$</td>
<td>thermal conductivity of solid metal ($W \cdot m^{-1} \cdot K^{-1}$).</td>
</tr>
<tr>
<td>$L$</td>
<td>characteristic dimension of the casting (m) (radius in the case of cylinder on one half of the edge for slabs).</td>
</tr>
<tr>
<td>$l$</td>
<td>effective mold length (m).</td>
</tr>
<tr>
<td>$m$</td>
<td>mass flow rate of water to the mold ($Kg \cdot s^{-1}$).</td>
</tr>
<tr>
<td>$P$</td>
<td>perimeter of the casting (m).</td>
</tr>
<tr>
<td>$Q$</td>
<td>rate of heat removal through the mold ($W$).</td>
</tr>
<tr>
<td>$S$</td>
<td>cross sectional area of mold ($m^2$).</td>
</tr>
<tr>
<td>$T$</td>
<td>average temperature of solid metal in the mold ($K$).</td>
</tr>
<tr>
<td>$t$</td>
<td>time ($s$).</td>
</tr>
<tr>
<td>$T_2$</td>
<td>average temperature of the solid at the point of complete solidification ($K$).</td>
</tr>
<tr>
<td>$T_{ave}$</td>
<td>average temperature of the cooling water circulating through the mold ($K$).</td>
</tr>
<tr>
<td>$T_f$</td>
<td>normal melting temperature ($K$).</td>
</tr>
<tr>
<td>$T_o$</td>
<td>reference temperature [e.g. that of the cooling medium ($K$)].</td>
</tr>
<tr>
<td>$T_{out}$</td>
<td>outlet and inlet temperatures of the cooling water to the mold ($K$).</td>
</tr>
<tr>
<td>$T_{w}$</td>
<td>average skin temperature of the casting inside the mold ($K$).</td>
</tr>
<tr>
<td>$T_{w}(z)$</td>
<td>skin temperature of casting at position $z$ in the mold ($K$).</td>
</tr>
<tr>
<td>$T_{ave}$</td>
<td>average solid temperature at position $z$ in secondary cooling section ($K$).</td>
</tr>
<tr>
<td>$u$</td>
<td>casting velocity ($m \cdot s^{-1}$).</td>
</tr>
<tr>
<td>$V$</td>
<td>$At/Av = \text{total volume of metal inside the mold (m}^3\text{)}$.</td>
</tr>
<tr>
<td>$V$</td>
<td>$\text{volume of metal in secondary cooling zone (m}^3\text{)}$.</td>
</tr>
<tr>
<td>$z$</td>
<td>position in the mold measured from meniscus (m).</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>density of the solid metal ($Kg \cdot m^{-3}$).</td>
</tr>
<tr>
<td>$\Delta z$</td>
<td>length of secondary cooling section (m).</td>
</tr>
<tr>
<td>$\Delta X_s$</td>
<td>average thickness of solid layer (m).</td>
</tr>
</tbody>
</table>

### REFERENCES