SHORT COMMUNICATION

The X_2 proof for weibull's distribution and the anisotropy of an electrical porcelain

P. Kittl*, J. H. C. Castro, W. J. Knapp*** and G. M. Camilo***

* Instituto de Investigaciones y Ensayes de Materiales (IDIEM), Universidad de Chile, Casilla 1420, Santiago, Chile.

** Universidade Federal de Sao Carlos, Departamento de Engenharia de Materiais, 13560 - Sao Carlos, S. P., Brasil.

*** Materials Science and Engineering Department, School of Engineering and Applied Science, University of California, Los Angeles, CA 90024, USA.

The present work utilized experimental data for strength in bending of a relatively large number (> 500) of specimens of an electrical porcelain. This note is concerned with the applicability of the Weibull model to these data.

Rectangular bars were formed by extrusion under vacuum, using a classical electrical porcelain body comprised of kaolin, ball clay, feldspar and silica †. The bars, after drying, were fired to 1200° C, and held at that temperature for three hours. The fired dimensions of the test bars were about 0.435 cm by 1.0 cm by 5.4 cm. The fired bars were loaded in three point bending, using a mechanical testing machine with a cross/head speed of 0.5 cm/min, to failure (Fig. 1). The apparent porosity of the bars (by ASTM C 20/46) was determined to be 0 %; closed pores were observed with examination of fracture surfaces by scanning electron microscopy.

As given by Weibull [1, 2], the cumulative probability of fracture in a non-uniform stress field is:

$$F(\sigma) = 1 - \exp\left\{-\frac{1}{V_o}\iiint_V \left(\frac{\sigma(x, y, z) - \sigma L}{\sigma_o}\right)^m dx\, dy\, dz\right., \quad (1)$$

where V_o is the unit volume (for this case 1 cm^3), $V = bhL$ is the sample's volume, σ_1 is the stress below which there is no fracture, and m and σ_o are specific parameters of the material. To calculate $F(\sigma)$, it is assumed that

$$\sigma_1 = o, \text{ and since } \sigma(x, y, z) = \frac{4\, xy\, \sigma}{Lh}$$

$$< \sigma = \frac{3}{2}\,\frac{PL}{bh^2},\ o < x < \frac{L}{2},$$

$$o < y < h/2,\ o < z < b, \text{ then:}$$

$$F(\sigma) = 1 - \exp\left\{-\frac{bhL}{2\,V_o\,(m+1)^2}\left(\frac{\sigma}{\sigma_o}\right)^m\right\} \quad (2)$$

The values of σ fell within 429 kgf/cm^2 (42.1 MPa) and 813 kgf/cm^2 (79.7 MPa); this interval was divided into 29 parts and the values for $F(\sigma)$ were calculated at points for $\sigma_i = (429 + i\Delta\sigma)$ kgf/cm^2, where $\Delta\sigma = 12.8$ and $i = 0.1, \ldots, 30$. Then $F(\sigma_i) = n_i^e/N$, where n_i^e is the number of samples broken experimentally at a stress equal or less than σ_i, and $N = 584$ (the total number of samples broken). Since the new variable.

$$\gamma_{n_i^e} = \ln\left\{\ln\frac{1}{1 - F(\sigma_i)}\right\} = \ln\left\{\ln\left(\frac{N}{N - n_i^e}\right)\right\} \quad (3)$$

can be linearly dependent of $\ln \sigma_i = \xi_i$ in accord with (2), it provides a useful evaluation to plot $\gamma_{n_i^e}$ as a function of $\xi_i = \ln \sigma_i$ which gives a straight line with some dispersion (Fig. 1). The equation for this straight line, obtained graphically, is:

$$\gamma_{n_i^t} = 14\,\xi_i - 92 \ldots \quad (4)$$

where n_i^t is the theoretical value of n_i there is an exact linear relation (4) between $\gamma_{n_i^t}$ and ξ_i.

On taking the natural log of equation (2) it can be found that:

$$\gamma_{n_i^t} = \ln\left(\ln\frac{1}{1 - F(\sigma_i)}\right) = \ln\left(\ln\frac{N}{N - n_i^t}\right) = m \ln \sigma_i + \ln\left(\frac{bhL}{2\,V_o\,(m+1)^2\,\sigma_o n}\right) \quad (5)$$

which allows us to obtain $\sigma_o \simeq 481$ kgf/cm^2 (47.2 MPa) and m = 14, once (5) is compared with (4). Now the theoretical values for n_i^t can be calculated from (5) as

$$n_i^t = N\left(\frac{e^{e^{\gamma_{n_i^t}}} - 1}{e^{e^{\gamma_{n_i^t}}}}\right) \quad (6)$$

where the differences $\Delta n_i^t = n_i^t - n_{i+1}^t$ and $\Delta n_i^e = n_i^e - n_{i+1}^e$, providing the theoretical and experimental frequency curves respectively. The applicability of the Weibull theory may be further evaluated by the X^2 «goodness of fit» test [3]. Using this test the value for X^2 is given by:

$$X^2 = \sum_{i=1}^{30}\frac{(\Delta n_i^t - \Delta n_i^e)^2}{\Delta n_i^t} \simeq 44.57 \ldots \quad (7)$$

† Overall oxides content of the fired body: 71.9 % SiO_2 22.9 %, Al_2O_3, 1.2 % CaO, 0.1 % MgO, 2.1 % K_2O, 0.6 % Na_2O, 0.7 % Fe_2O_3, 0.5 % TiO_2 (by weight).

This value falls in the interval:

$$X^2_{0{\cdot}975} = 45.7 > \chi^2 = 44.7 > \chi^2_{0{\cdot}025} = 16$$

for 29 degrees of freedom (number of intervals minus 1). A gaussian distribution was also used, but $X^2 = 180$ eliminated this distribution.

Thus, it is concluded that the Weibull's function (1) gives a representation of the experimental results with a X^2 test corresponding to the 95 % significance level, giving a better correlation than provided by a gaussian function.

In order to confirm this conclusion a new batch of 55 samples was tested (Fig. 1). But now the sample with b = 0.45 cm, h = 1 cm and L = 4.95 cm. Formula (5) schows that m is the same with this new configuration in the test, but experiments give m = 4.67 so it is concluded that must be a strong anisotropy in the sample. Investigation in order to check this is at present being experimentally made.

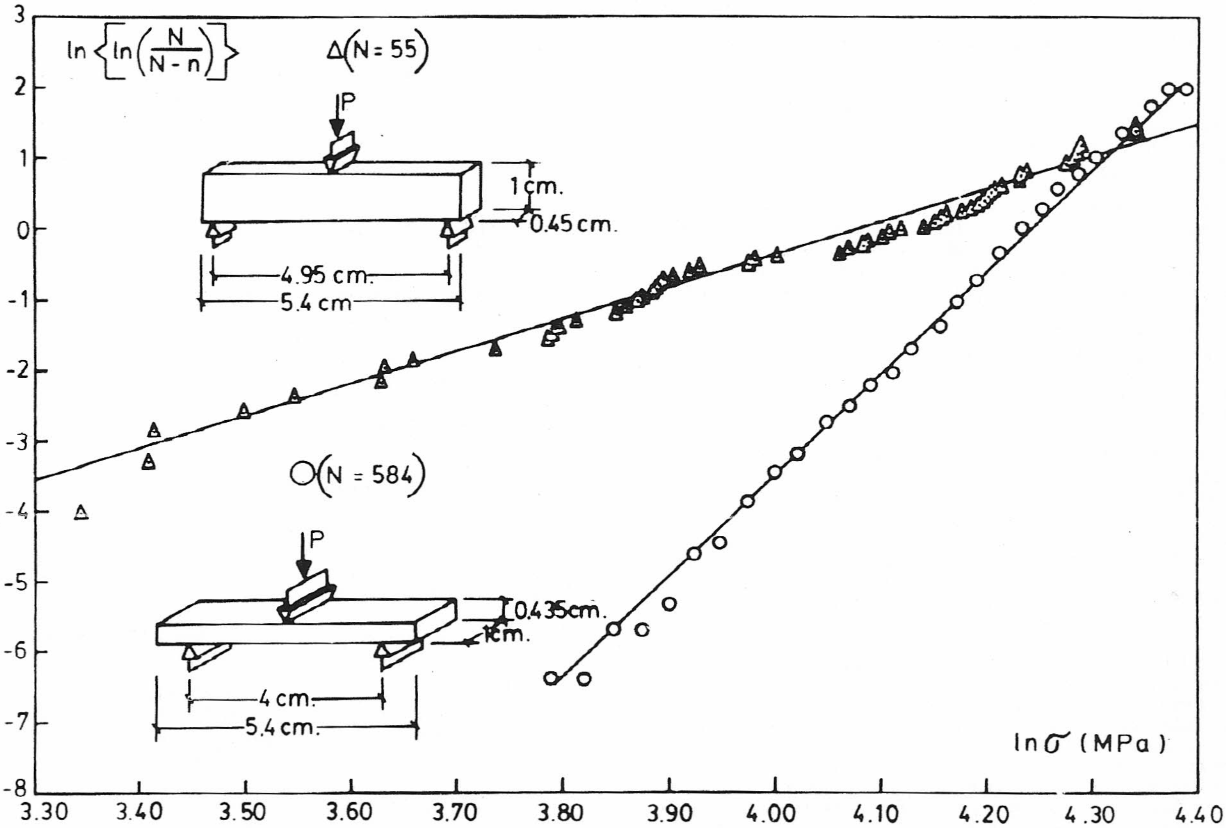

Fig. 1. Dimensions of the samples and Weibull's plot.

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