SPIN-ORBIT EFFECTS ON SINGLE PARTICLE TRANSITIONS IN QUANTUM WELLS AND RINGS

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SPIN-ORBIT EFFECTS ON SINGLE PARTICLE TRANSITIONS IN QUANTUM WELLS AND RINGS

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Resumen
Se considera un anillo cuántico con ancho finito, sujeto a la acción de un campo magnético externo aplicado en la dirección transversal cuando ambos tipos de interacción espín-orbita Rashba-Bychkov y Dresselhauuss son tenidas en cuenta. Para estudiar las transiciones de una sola partícula, se extendió el método propuesto por Lipparini et al. para pozos cuánticos al caso de confinamiento unidimensional fuerte. Expresiones analíticas para los estados de una partícula cuasi up y cuasi down y sus energías han sido obtenidas. El nivel de transición de una sola partícula inducida por un campo electromagnético ac ha sido considerado y los elementos de matrix concernientes a estas transiciones relacionadas con la resonancia ciclotrónica (qup-qup y qdown-qdown) y las transiciones spin-flip han sido calculadas.

Palabras Claves: Interacción Rashba-Bychkov, Interacción Dresselhauuss, Anillo cuántico

Abstract

It is considered a quantum ring with finite width, subjected to the action of an external magnetic field applied in the transversal direction when both Rashba-Bychkov and Dresselhauuss interactions are taken into account. In order to study the single particle transitions, we extend the method proposed by Lipparini et al. for quantum wells, to the case of strong 1D confinement. Analytical expressions for quasi up and quasi down single particle states and energies have been obtained. Single particle level transitions induced by ac electromagnetic fields have been considered and the matrix elements concerning the allowed transitions related to cyclotron resonance (qup-qup and qdown-qdown) and spin-flip transitions have been calculated.

Key words: Rashba-Bychkov interaction, Dresselhauuss interaction, Quantum Rings

1. INTRODUCTION

Considerable attention has been devoted to research concerning the control and manipulation of the spin-orbit (SO) interaction in semiconductor nanostructures [1-7]. In this sense, it is important to understand the way how the application of external fields can affect the electronic properties of particles under the conditions of strong confinement. In this report we consider a quantum ring with circular cross section, subjected to the action of an external magnetic field applied in the transversal direction when both Rashba-Bychkov and Dresselhauuss interactions are taken into account. In order to study the single particle transitions, we extend the method proposed by Lipparini et. al for quantum wells [5], to the case of a finit width ring with strong 2D confinement.

This work is organized as follows. In Sec. 2 we present the general formalism for the single-particle (sp) Hamiltonian. These results are used in Sec. 3 to study the transitions caused by an external electromagnetic field.

2. THEORETICAL FRAMEWORK

We consider an electron of charge $e$ and effective mass $m$ in a (2D) ring in the plane $z = 0$, centered on the $z$ axis, with external radius $\rho_{Ex}$ and inner radius $\rho_{In}$ under an uniform magnetic field $B$ normal to the plane of the ring sample. The single particle Hamiltonian $H_0$
\[ H = \frac{p^2}{2m} + \frac{1}{2} g \mu_B B \sigma_z + \frac{i}{\hbar} \left( \sigma_x P_y - \sigma_y P_x \right) + \frac{i}{\hbar} \left( \sigma_z P_x - \sigma_x P_z \right) \]

contains the following terms: (i) kinetic; (ii) Zeeman; (iii) Rashba; (iv) Dresselhauß, where \( p = -i\hbar \nabla + (e/c) A \), \( A = (1/2) B \rho \), \( g \) is the effective gyromagnetic factor, \( \mu_B \) is the Bohr magneton and \( \sigma_z \) \( (i=x, y, z) \) are the Pauli matrices. Schrödinger equation \( H_0 |\psi\rangle = \hbar \omega_0 c |\psi\rangle \) can be solved by expanding the components \( |\phi_i\rangle, |\phi_f\rangle \) of the two-component spinor \( |\psi\rangle \) in terms of the electron states in absence of spin-orbit effects as \( |\phi_{m\alpha}(\xi, \phi)\rangle = \Sigma_{\alpha, m} \exp(i m \phi) R_{m\alpha}(\xi) \), where \( \alpha = 1, 2, m = 0, 1, \pm 2, \ldots, \)

\( R_{m\alpha}(\xi) = [M(\alpha, m, 1, \pm 1, \xi) \pm \Lambda U(\alpha, m, m, m, 1, \xi)] \xi^m |\xi|^{1/2} \exp(-\xi^2/2) \)

are the solutions of the Kummer equation. We obtain

\[
\left( \frac{\xi^2 - \epsilon}{\hbar \omega_0} - \epsilon \right) c_{1m} + \tilde{\lambda}_x \tilde{A}_{2m} c_{2m+1} - i \tilde{\lambda}_x \tilde{A}_{2m+1} c_{2m-1} = 0
\]

\[
\left( \frac{\xi^2 - \epsilon}{\hbar \omega_0} - \epsilon \right) c_{2m} - \tilde{\lambda}_x \tilde{A}_{1m} c_{1m+1} + i \tilde{\lambda}_x \tilde{A}_{1m+1} c_{1m-1} = 0
\]

where \( \tilde{\lambda}_x = \left( 2 \sqrt{\frac{\hbar}{\omega_0}} \right) \lambda_x A_{21} \)

\[
\frac{\epsilon\bar{\omega}_0}{\hbar \omega_0} = \tilde{\lambda}_x = \pm \frac{1}{2} \left( m + m + 1 \right)
\]

\[
A_{2m} = \int R_{m\alpha}(\xi) d\xi N_{\alpha}^{-1} N_{\alpha} = \int \sqrt{2} \xi R_{m\alpha}(\xi) d\xi
\]

\[
\tilde{R}_{m}(\xi) = 2\sqrt{\xi} \frac{\partial R_{m\alpha}(\xi)}{\partial \xi} = \left( \frac{m}{\sqrt{\xi}} + \sqrt{\xi} \right) R_{m\alpha}(\xi)
\]

Following Lipparrini et al. [5], analytical expressions for quasi up and quasi down single particle states and energies are obtained in the limit \( \tilde{\lambda}_x < \xi \) by considering that for each state \( |\phi_{m\alpha}(\xi, \phi)\rangle \) spin orbit interaction is allowed to couple it only with states \( |\phi_{m+1\alpha}(\xi, \phi)\rangle \). For the case of quasi up (qup) states, we obtain

\[
\{ \langle \phi_f | \phi_i \rangle \} = \sum_{m} \left( c_{1m} |\phi_{m+1}\rangle + c_{1m} |\phi_{m+1}\rangle \right)
\]

\[
c_{wp} = -\alpha + \frac{1}{4} g + \frac{1}{2} (m + m + 1) - \frac{2}{g} \tilde{A}_{2m} (\tilde{A}_{2m+1} + \tilde{A}_{2m+1}) \]

where

\[
c_{1m} = 1 - 2 \tilde{A}_{2m} (\tilde{A}_{2m+1} + \tilde{A}_{2m-1}) / g, \]

\[
c_{2m+1} = -2i \tilde{A}_{2m} (\tilde{A}_{2m+1} - \tilde{A}_{2m-1}) / g
\]

Similar expressions for quasi down (qdown) states are also obtained.

\[
\{ \langle \phi_f | \phi_i \rangle \} = \left( c_{1m-1} |\phi_{m+1}\rangle + c_{1m+1} |\phi_{m-1}\rangle \right)
\]

\[
c_{wd} = -\alpha - \frac{1}{4} g + \frac{1}{2} (m + m + 1) + \frac{2}{g} \tilde{A}_{2m} (\tilde{A}_{2m-1} + \tilde{A}_{2m+1}) \]

where

\[
c_{2m} = 1 - 2 \tilde{A}_{2m} (\tilde{A}_{2m-1} + \tilde{A}_{2m+1}) / g, c_{1m-1} = 2i \tilde{A}_{2m} (\tilde{A}_{2m-1} - \tilde{A}_{2m+1})
\]

\[
c_{1m+1} = -2 \tilde{A}_{2m} (\tilde{A}_{2m-1} - \tilde{A}_{2m+1})
\]

We can use the preceding results to study the transitions induced in the system by the interaction with a left circular-polarized electromagnetic wave propagating along the z direction, i.e., perpendicular to the plane of motion of the electrons, whose vector potential is \( A(t) = 2A \cos \theta, A \) in the dipole approximation \( \theta = 0 \).

The single particle (sp) interaction Hamiltonian

\[
j \cdot A/c + g \mu_B (\nabla \times A) \]

where \( j = e \kappa \sqrt{\epsilon} \) reads

\[
h_{int} = \frac{eA}{\kappa \sqrt{\epsilon}} [v_x \exp(i \theta) + v_y \exp(-i \theta)]
\]

\[
+ \frac{g \mu_B A \rho}{2} \left[ \sigma_x \exp(i \theta) + \sigma_y \exp(-i \theta) \right]
\]

where \( v_x = \frac{p}{m} \pm i \frac{\lambda_x}{\hbar} \sigma_x \pm \frac{\lambda_y}{\hbar} \sigma_y \). The Hamiltonian \( h_{int} \) can be rewritten as

\[
h_{int} = \frac{eA}{\kappa \sqrt{\epsilon}} [\sigma_x \exp(i \theta) + \sigma_y \exp(-i \theta)]
\]

\[
+ \frac{1}{2} A \rho [\sigma_x \exp(i \theta) + \sigma_y \exp(-i \theta)]
\]

The operators \( \alpha, \gamma \) acting on the spinor \( |\phi_{m\alpha}\rangle \) are
\[ \alpha_+ = \left( \frac{1}{m} P \frac{i 2 \lambda_d}{\hbar}, \frac{1}{m} P \frac{2 \lambda_d}{\hbar} \right) \], \quad \alpha_- = \left( \frac{1}{m} P \frac{i 2 \lambda_d}{\hbar}, \frac{1}{m} P \frac{2 \lambda_d}{\hbar} \right) \quad (10) \]

3. RESULTS AND DISCUSSION

Due to the fact that \( g < 0 \), the lowest energy level in GaAs rings is the quasi up one at an energy, containing both Rashba and Dresselhauuss contributions, which is in contrast with the behavior of such level in quantum wells, containing only the Rashba contribution [5].

In Fig.1, we want to show that for a given value of the strength of Dresselhauuss interaction (\( \lambda_{D} \)), the splitting of\( \delta E = \left[ E_{\uparrow} - \left( E_{\downarrow} / \hbar \right) \right] \) of the single particle energy due to spin-orbit effects increases with the square of the intensity of the Rashba term. It is seen that spin-orbit effects do not alter the period of Aharonov-Bohm oscillations in 1D quantum rings, but it is observed a shift toward the left of the degenerated points. It is important to remark that there exists a set of values of the applied magnetic field for which the shift of the SO oscillations disapears. This fact can motivate the search of new electron transport mechanisms as it is discussed by Fabrício et. al [6].

In Fig.2, we illustrate the role of the Zeeman effect on the energy of quasi up (qup) and quasi down (qdown) states as a function of the magnetic field for a quantum ring of finite width of dimensions \( \rho_{ln} = 5a_{B} \) and \( \rho_{Es} = 20a_{B} \). It is seen that the Zeeman effect increases the energy of the qdown state for \( m=0,1,-1 \) (dashed lines), and decreases the energy of the qup state for \( m=0,1,-1 \) (thick lines) with respect to the corresponding states in the absence of Zeeman effect (thin lines).

In Fig.3, we show the splitting of the energies for the qup and qdown states due to the SO interaction as a function of the magnetic field for a quantum ring of finite width of dimensions \( \rho_{ln} = 5a_{B} \) and \( \rho_{Es} = 20a_{B} \). It is seen that for the qup states (Fig3a) the energy splitting increases linearly with the magnetic field for \( m=0,1,-1 \), and for the qdown states (Fig.3b) it decreases linearly with the magnetic field for \( m=0,1,-1 \). The inset to Fig.3a shows a zoom of the qup and qdown energies for a wider range of magnetic fields. It is noted that the corresponding monotonous behaviour of the splitting of the ground state energy with the magnetic field is in contrast with the corresponding oscillatory behaviour of such splitting in 1D quantum rings.

\[ \delta E = \delta \lambda_{D} = 0.01 \]

\[ \delta \lambda_{D} = 0.025; \delta \lambda_{D} = 0.01 \]

\[ \delta \lambda_{D} = 0.03; \delta \lambda_{D} = 0.01 \]

Figure 1. Splitting of the lower single particle quasi-up energies in GaAs 1D quantum rings \( \delta E \) (in units of \( h^2/2m^{*}a_{B}^{3} \)) as a function of the magnetic flux \( \Phi \) (in units of the quantum of magnetic flux \( \Phi_{0} \)) for a fixed value of the strength of Dresselhauuss interaction (\( \lambda_{D} = 0.01 \)) and for: (a) \( \lambda_{D} = 0.0 \), (b) \( \lambda_{D} = 4 \lambda_{D} \). Calculations are performed for \( g = -0.44 \).
We consider next several useful examples of spin matrix elements involving the operators $\alpha$ for a 1D quantum ring, which are proportional to $v_+$ and $\sigma_-$, and the qup and qdown sp states of Eqs. 4 and 6.

The qup-qdown and qdown-qup transitions with energy $E_n^d - E_n^u$ have the matrix element

$$
\delta E \propto \lambda D \left( \frac{h^2}{2m} \right) R \left( n^2 + \frac{1}{4} \right).
$$

Note that the transition matrix element is linear in $\lambda D$, and that in the presence of the Rashba

![Figure 2. Splitting of single particle energies of a charge carrier in a quantum ring of finite width of dimensions $\rho_{In} = 5a_B$ and $\rho_{Ex} = 20a_B$ as a function of the applied magnetic field. The thin continue lines correspond to states in the absense of Zeeman effect.](image)

![Figure 3. Spin-orbit splitting on the energy (in units of $(h^2/2m\rho_{Ex})*10^{-4}$) of some quasi up (figure 3a) and quasi down states (figure 3b) for a confined carrier in a quantum ring of finite wide as a function of the magnetic field.](image)
interaction alone, \( \alpha \) causes no spin-flip transition, which is in agreement with the result obtained by Lipparini et al. for quantum wells[5]. Additionally, spin-flip excitations are also possible at an energy \( E_{n+1} - E_n^+ \) with matrix element \( \langle n, \sigma | \sigma, n \rangle \approx 2 \), which is also in agreement with similar result for quantum wells.

4. CONCLUSION
We have studied the influence of SO interactions on the electronic spectrum of confined carriers in quantum rings. It was shown that the magnetic field behavior of the ground state energy differs considerably in 1D rings when compared with rings with finite width. It was found that in 1D quantum rings the lowest energy state depends on both Rashba and Dresselhauuss terms, which differs considerably with the behaviour of such state in quantum wells, where only the Rashba contribution is present. It was found also that the shift of the SO oscillations in 1D rings can disappear for a special set of values of the applied magnetic field. Finally, it was demonstrated that Rashba and Dresselhauuss contributions can control the transitions between energy states involving spin flip transitions.

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6. REFERENCES